Visualization of Refractive Index and Reflectance of Sodium Metal Based on Drude's Theory Using MATLAB as an Alternative for Optics and Astrophysics Lectures

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Abstract – This study aims to simply examine the relationship between optics and astrophysics by visualizing the refractive index and reflectance of sodium metal based on Drude's theory using MATLAB. Sodium metal is used in this study because it is an alkaline metal found on Earth which is a life-supporting element. MATLAB is used to visualize the refractive index and reflectance of sodium metal based on Drude's theory because it is one of the software that is often used by students in supporting optics and astrophysics courses. This research presents a simple way for students to visualize the refractive index and reflectance of sodium metal based on Drude's theory with the help of MATLAB. Visualization of the refractive index and reflectance of sodium metal based on Drude's theory begins by describing the equations of the refractive index and reflectance based on Drude's theory. We realized the visualization of these equations in graphical form with the help of MATLAB.

In this study, students were able to visualize the refractive index and reflectance of sodium metal based on Drude's theory using MATLAB by varying the frequency of sodium plasma and sodium attenuation frequency.

Keywords – astrophysics, Drude's theory, optics, MATLAB, sodium.

1. Introduction

The scientific branch of physics varies, and it consists of several branches of science such as mechanics, optics, thermodynamics, electrodynamics, astrophysics, and modern physics. Each of these branches of physics is related to one another when solving physical phenomena that occur in life, as with astrophysical phenomena that cannot be separated from optical studies [1]. Physical phenomena that occur in everyday life include astrophysical and optical studies as well as the phenomenon of reflection and refraction of sunlight which have different characteristics when viewed from the planets Mars and Earth [2]. This, of course, can be an important study for the advancement of physics, especially optics and astrophysics.

The relationship between the two studies can provide a new understanding of the nature of light reflection and refraction. Besides, the existence of physics studies that are connected certainly makes it easier for innovators in the field of optical and astrophysical knowledge and technology to develop sensitive equipment capable of overcoming the strong radiation emitted by sunlight [3]. The existence of physics studies that are linked to each other, it can increase curiosity to know material objects that exist on planets other than Earth. Thus, the study of the branches of physics that are connected can make it easier to interpret physical phenomena and provide more solutions in developing physical technologies that are beneficial to the life of the universe.
Although the study between the branches of physics provides many benefits, in practice it takes a long time and requires a stronger ability to understand the combined material [4]. This is certainly one of the obstacles in studying several branches of physics at the same time. Besides, the problem of equitable education is also one of the factors that cause difficulty in studying physics scientific branches simultaneously [5].

There are physics courses conducted to students on optics and astrophysics material that some students find difficult [6]. Moreover, if two branches of physics are delivered simultaneously in lectures, it certainly allows new problems to emerge such as misconceptions in students and less in-depth lectures. However, these problems can be overcome with several solutions, such as raising awareness of the importance of innovation and collaboration to continue to develop physics science through simultaneous studies of optics and astrophysics [7]. This requires more time from students and education so that they can find out the relationship between the branches of physics. One of the efforts that can be made by educators is by implementing optical and astrophysical concepts with habits that are often done by students such as technology [8]. However, the technology integrated into the combined optics and astrophysics course uses technology that is inexpensive, easy to obtain, accessible, and operated by students.

One of the integrations of the concepts of optics and astrophysics can be done with the help of technology that is cheap, easy to obtain, accessible, and operated by students such as MATLAB. However, in integrating the concepts of optics and astrophysics, of course, students are given the flexibility to construct the visualization of optical and astrophysical concepts so that they can interpret them more deeply. Most optical and astrophysical visualization concepts are in the form of a set of teaching aids to help visualize basic concepts of optics and astrophysics as well as the concept of optical devices such as telescopes [9].

However, visualization for the concept of optics-astrophysics specifically as well as the optical properties of material objects on planets in the solar system has not been studied much [10]. Moreover, the form of optical-astrophysical concept visualization is in the form of physical objects which are not efficient and effective when used by large numbers of students. Besides, the form of visualization of the optical-astrophysical concept is not integrated with the help of technology which causes its users to be limited and cannot be used anywhere and anytime [11]. Unlike the case with the visualization of the optical-astrophysical concept that integrates technology such as MATLAB which can be used by students and educators anytime and anywhere. Thus, with the help of technology such as MATLAB, which is used in visualizing the concepts of optics-astrophysics, it is hoped that optics and astrophysics courses can be integrated and run effectively.

Through activities that visualize the optical-astrophysical concepts integrated with MATLAB, of course, it can make it easier for educators and students to develop optical and astrophysical science [12]. Besides, with the existence of activities that visualize the optical-astrophysical concept integrated with MATLAB, it can also provide alternative lectures easily and efficiently. Furthermore, this statement is also supported by the results of research conducted by Zhao et al. [13] that visualizing the optical-astrophysical concept of the design and analysis of a tertiary mirror system from a thirty-meter telescope in the conceptual phase can be done with interactive visualization results with the help of MATLAB. Furthermore, MATLAB is computational software that is used to easily visualize a mathematical equation of optical and astrophysical phenomena that does not require an internet connection [14]. Thus, the existence of activities that visualize the optical-astrophysical concepts that are microscopic to macroscopic which cannot be directly observed by the five human senses is required in optics-astrophysics courses.

MATLAB software is software that has not been installed directly on the computer, so it is necessary to install it first if we are going to operate it. MATLAB is graphical computation software that is used to visualize mathematical equations with the results of visualizing two- and three-dimensional graphs [15]. Using MATLAB, users can manipulate a matrix, plot mathematical functions, and equations, implement an algorithm and create a user interface. MATLAB is also computational software that can analyze simple to complex algebraic equations. MATLAB can be used in supporting optical and astrophysical research activities, especially in analyzing and visualizing mathematical equations of complex optical and astrophysical phenomena [16]. The implementation of MATLAB in optics and astrophysics courses can support simple visual constructs of every optical and astrophysical equation [17]. The implementation of MATLAB in optics and astrophysics courses also has a positive impact on students' abilities. This is by the results of the research which revealed that optics and astrophysics courses that integrate graphic visualization produced by MATLAB can improve students' visual representation skills [18]. Mathematical and graphic representation abilities in students can also be developed by applying optics and astrophysics courses assisted by MATLAB [19].
After students have good visual representation skills, they can use MATLAB to visualize mathematical equations of other optical and astrophysical phenomena creatively and innovatively [20]. This is because MATLAB is graphical computing software capable of visualizing optical and astrophysical phenomena and their equations from simple to complex in a complete and easy to understand manner. Besides, MATLAB is also software that has been widely used by educators and students in helping them understand abrupt optical and astrophysical materials whose symptoms cannot be directly captured by the five senses [21]. Furthermore, several findings reveal the application of MATLAB software in optics and astrophysics courses. Therefore, with the various uses of MATLAB in optics and astrophysics courses and the rare visualization of the refractive index and metal reflection of sodium based on Drude's theory using MATLAB's assistance, it is necessary to innovate the visualization development using MATLAB.

2. Theory

Drude's free-electron theory assumes that the valence electrons of atom areas if free. Since the free electrons are not attached to any atom, there is no resonant recovery force. However, with an applied electric field, free electrons will experience an electric force [22]. Furthermore, these electrons experience a collision with the scattering time given by \( t \). Considering the scattering process, Drude's model for conductivity can be generalized to include the effect of alternating current (AC) by applying Newton's second law as shown in equation (1) below.

\[
m \left( \frac{d\vec{v}}{dt} + \frac{v}{\tau} \right) = -e\vec{E}(t) = -e\vec{E}_0 e^{-i\omega t} \quad (1)
\]

Equation (1) shows that the velocity-dependent damping term has been introduced and \( t \) is the relaxation time. Besides, there is a component in the form of a complex notation to describe a time-dependent electric field as shown in equation (2) below.

\[
\vec{v}(t) = \vec{v}_0 e^{-i\omega t} \quad (2)
\]

Equation (2) can also be described by separating the amplitude components as shown in equation (3) below.

\[
\vec{v}_0 = -\frac{e}{m} \left( -\frac{\vec{E}_0}{-i\omega + 1/\tau} \right) \quad (3)
\]

The next step is to define the time-dependent current density vector as shown in equation (4) below.

\[
\vec{j}(t) = \vec{j}_0 e^{-i\omega t} \quad (4)
\]

After equation (4) is determined, what needs to be done is to combine equation (4) with the current density equation as shown in equation (5) below.

\[
\vec{j}_0 = -ne\vec{v}_0 = \sigma(\omega)\vec{E}_0 \quad (5)
\]

Furthermore, express the frequency-dependent AC conductivity equation as can be shown in equation (6) below.

\[
\sigma(\omega) = \frac{\sigma_0}{1 - i\omega\tau} \quad (6)
\]

Equation (6) can also be described by separating the components as shown in equation (7) below.

\[
\sigma_0 = \varepsilon_0 \omega_p^2 \tau \quad (7)
\]

Next is to express the plasma frequency equation as shown in equation (8) below.

\[
\omega_p^2 \equiv \frac{ne^2}{m\varepsilon_0} \quad (8)
\]

Equation (8) represents the oscillation frequency of the neutral gas of charged particles known as plasma. Both metals and semiconductors can be physically modelled as a collection of positive ions and the same free electron charge, namely plasma [23]. Furthermore, common metals have a typical plasma energy value in the range of 3-17 eV. The relaxation time is usually out of order \( 10^{-14} \) seconds and the damping constant arising from the relaxation time is in the order of 0.1 eV. Equation (7) can be used to interpret the conductivity of the complex by multiplying the numerator and denominator by the complex conjugate \( 1 + i\omega\tau \), so that the following equation (9) is obtained.

\[
\sigma(\omega) = \left( \frac{ne^2\tau}{m} \right) \frac{1 + i\omega\tau}{1 + \omega^2\tau^2} \quad (9)
\]

In equation (9) it is known that as \( \omega \to 0 \), \( \Re\sigma(\omega) \) reduced to the Drude conductivity formula (or DC conductivity) as shown in equation (10) below.

\[
\sigma = \frac{ne^2\tau}{m} \quad (10)
\]

At the low-frequency limit, \( \Im\sigma(\omega) \to 0 \) as \( \omega \to 0 \). The dielectric constant of the complex and the conductivity of the complex AC are related to each other [24]. This can be shown by using the fourth Maxwell equation and replacing the frequency-dependent electric field equation on the right side of the equation so that the following equation (11) is obtained.

\[
\vec{\nabla} \times \vec{B} = \mu_0 \frac{\partial}{\partial t} \varepsilon(\omega)\vec{E}(t) \quad (11)
\]
Equation (10) can also be described by separating the components as shown in equation (11) below.

$$\varepsilon(\omega) = \varepsilon_0 + \frac{i\sigma(\omega)}{\omega}$$  

(11)

Based on the elaboration of equation (11), it can be concluded that optical measurement $\varepsilon(\omega)$ equivalent to AC conductivity measurement. Therefore, the next step is to substitute equation (9) into equation (11), so that the frequency-dependent relative dielectric function equation is obtained $\varepsilon_r(\omega)$ as shown in equation (12) below.

$$\varepsilon_r(\omega) = 1 - \frac{\omega_p^2}{\omega(\omega + i/\tau)}$$  

(12)

Next, extract the real and imaginary parts of Equation (12) and can be written as shown in equation (13) below.

$$3m[\varepsilon_r(\omega)] = \frac{\omega_p^2\gamma}{\omega(\gamma^2 + \omega^2)}$$  

(13)

Based on equation (13) it is known that $\gamma \equiv 1/\tau$ acts as the frequency of attenuation. Next, analyze equation (13) in the case of a weakly damped system by setting $1/\tau \to 0$ as shown in equation (14) below.

$$\varepsilon_r(\omega) = 1 - \frac{\omega_p^2}{\omega^2}$$  

(14)

It is known that the dielectric function of the complex and the refractive index of the complex are related. Besides, the reflectance equation can also be applied to the metal-air interface [25]. In this case, $n_1 = 1$ (real, air) and $n_2 = \tilde{n} + iK$ (imaginary, metal). Therefore, the equation for plasma reflectivity ($R$) can be shown as equation (15) below.

$$R = \left| \frac{\tilde{n} - 1}{\tilde{n} + 1} \right|^2$$  

(15)

When $\omega < \omega_p$, then $\tilde{n}$ imaginary and $\omega > \omega_p$, then $\tilde{n}$ real and positive. In $\omega = \omega_p$, then the refractive index is equal to zero. To $\omega < \omega_p$, $R = 1$ and dropping for $\omega > \omega_p$. In simple terms, this implies that the reflectivity of free electron gas (plasma) is 100% up to the frequency of the plasma. So, most metals spark up to plasma frequencies and will reflect visible and infrared light [26]. This is the reason why metals such as silver and aluminium have been used for centuries to make mirrors. Beyond the plasma frequency, some light can be transmitted through metals [27]. This implies that the metal will eventually become transparent for frequencies that extend to the ultraviolet $\omega > \omega_p$ and this phenomenon is known as the ultraviolet transparency of metals. At first glance, it should be noted that some metals such as gold and copper appear colored because they selectively absorb certain wavelength ranges [28]. This effect is in addition to the plasma reflectivity effect.

3. Method

Visualization of the refractive index and reflection of Sodium metal based on Drude's theory was carried out in this article using the help of MATLAB software. The purpose of using MATLAB is to determine the visualization of the refractive index and reflection of sodium metal based on Drude's theory. With this, a way to visualize the refractive index and reflection of sodium metal based on Drude's theory is obtained which is easy to do in helping lectures in optics and astrophysics. MATLAB software is used to help visualize the refractive index and reflection of sodium metals based on Drude's theory because it can strengthen mathematical understanding of the concepts of optics and astrophysics and produce smooth visualizations [29]. Besides, MATLAB also has a feature to visualize graphics with a variety of shape choices, both two-dimensional and three-dimensional, so that it can be used to plot the visualization of the refractive index and metal reflection of Sodium based on Drude's theory based on equation (15). In simulating the refractive index and reflection of sodium metal based on Drude's theory with the help of MATLAB software, several steps need to be taken to obtain the visualization.

The steps that need to be done are starting with determining the plasma frequency and the damping frequency of the sodium metal which will be inputted in equation (15). This is done so that the mathematical equations of the refractive index and metal reflection of Sodium based on Drude's theory that is input in the MATLAB workspace can be processed properly and produce smooth visualization. Thus, the syntax that is inputted into the MATLAB workspace to visualize the refractive index and metal reflection of Sodium based on Drude's theory which contains equation (15) can be presented as follows.

% This script plots the refractive index (n), extinction coefficient % kappa, and the reflectance derived within the Drude theory. % The plasma frequency and damping % parameters are for Na (metal). % By Himawan Putranta omp = 5.914; % Na plasma frequency (eV) hgamma = 0.0198512; % Na damping frequency (eV) % Function definitions
% Frequency dependent real part of the dielectric function.
realpart = @(w) 1 - omp^2/(hgamma^2 + w^2);

% Frequency dependent imaginary part of the dielectric function.
impart = @(w) (omp^2)*hgamma/((w)*(w^2 + hgamma^2));

% Refractive index (n) and extinction coefficient (κ)
nw = @(w) sqrt(0.5*(sqrt((1+realpart(w))^2 + impart(w)^2)) + realpart(w));
kappaw = @(w) sqrt(0.5*(sqrt((1+realpart(w))^2 + impart(w)^2)) - 1 - realpart(w));

% Reflectance
refl = @(w) ((1 - nw(w))^2 + kappaw(w)^2)/((1 + nw(w))^2 + kappaw(w)^2);

% Plots
figure1 = figure;
axes1 = axes('Parent',figure1,'YMinorTick','on','XTick',[0 1 2 3 4 5 6 7 8 9 10 11 12],'
XMinorTick','on');
ylim([0,2]);
hold(axes1,'all')
hold on;
fplot(@(w)nw(w),[0,10],'LineWidth',2,'--k');
fplot(@(w)kappaw(w),[0,10],'LineWidth',2,'-or');
fplot(@(w)refl(w),[0,10],'LineWidth',2,'-b');
% Create xlabel
xlabel('Energy,(eV)','FontSize',16,'FontName','Times New Roman');
% Create ylabel
ylabel('Optical Constants and Reflectance', 'FontSize',16,'FontName','Times New Roman');
% % Create legend
legend1 = legend('n(\omega)','\kappa(\omega)','R(\omega)');
set(legend1,'FontSize',14,'FontName','Times New Roman','show');
hold off;

The workspace display of the MATLAB software that has been inputted with equation (15) is needed to visualize the refractive index and reflection of sodium metal based on Drude's theory can be shown in Figure 1 below.

![Figure 1. Display of MATLAB workspace for visualizing the refractive index and reflection of sodium metal based on Drude's theory](image)

After inputting equation (15) which is needed to visualize the refractive index and reflection of sodium metal based on Drude's theory as shown in Figure 1, the next step is to visualize the refractive index and reflection of Sodium metal based on Drude's theory. The step that needs to be done to bring up the visualization of the refractive index and reflection of sodium metal based on Drude's theory in MATLAB is by clicking the Run option on the Editor taskbar as shown by the red circle in Figure 2 below. After the visualization of the refractive index and metal reflection of Sodium based on Drude's theory appeared, the next step was to analyze the visualization.
4. Results and Discussion

In this article, we show the process of visualizing the refractive index and reflection of sodium metal based on Drude's theory using the help of MATLAB. Visualization of the refractive index and reflection of sodium metal based on Drude's theory was carried out by varying the frequency value of sodium metal plasma of 5.914 eV and the damping frequency value of sodium metal of 0.0198512 eV. One of the reasons why sodium metal is used in the study of this article is because sodium is an alkaline metal that is needed to support life in the world [30]. Besides, variations were made to the plasma frequency and attenuation frequency of sodium metal to know the input and output processes in the form of visualization of the refractive index and reflection of Sodium metal based on Drude's theory using MATLAB software which is easier and more interactive when used in optical and astrophysics. Meanwhile, the results of visualization of the refractive index and reflection of sodium metal based on Drude's theory using MATLAB can be shown as in Figure 3 below.

Based on Figure 3, the visualization of the refractive index of sodium metal based on Drude's theory forms a downward exponential graph. However, after the energy is at a value of 4.2 eV, the refractive index of sodium metal based on Drude's theory increases with the increase in energy. In general, the visualization of the refractive index of sodium metal based on Drude's theory forms an exponential graph. Furthermore, the reflection of sodium metal based on Drude's theory decreases at a
stable value, but when the energy is at the value of 4.2 eV, the reflection of the sodium metal falls until the value is close to zero. Meanwhile, the Kappa coefficient as the extinction coefficient shows a drastically decreasing curve, but when the energy value is 4.2 eV.

The curve shows the stability of the Kappa value is close to zero. Based on the results of the visualization, in general, the refractive index, Kappa coefficient, and reflection of sodium metal based on Drude's theory show an exponential curve. Besides, the energy or frequency of sodium metal at the value of 4.2 eV becomes the reference point when the refractive index curve, Kappa coefficient, and sodium metal reflection occur. This is one of the main characteristics of sodium metal in terms of optics and astrophysics. With the visualization that shows the characteristics of sodium metal, sodium metal can be one of supporting materials for making components in telescopes such as in the lens [31].

Sodium metal also has a role in astrophysical studies. Metal sodium is identified by the D spectral line in the interstellar medium. Although sodium metal has a high evaporation temperature, its abundance in the planet Mercury's atmosphere made it possible to detect it by Potter and Morgan using high-resolution earth station spectroscopy. Metal sodium has been detected in at least one comet that astronomers have detected. This was proven when astronomers watched Comet Hale-Bopp which has a sodium tail containing neutral atoms (not ions) and reaches up to 50 million kilometres behind its head [32]. Furthermore, this visualization of the refractive index and metal reflection of sodium based on Drude's theory has an important purpose in the field of optics and astrophysics education.

In the field of optics, the benefits obtained from this visualization activity are to find out the optical characteristics of material objects on Earth and on other planets scattered in the Milky Way galaxy. Through this activity, it can also be seen the advantages and disadvantages of material objects that can be used as loading tools to observe space objects such as telescopes [33]. In the field of astrophysical research, the benefits obtained from this visualization activity are to find out whether there are similarities between the material on Earth and other planets and to find out the process of the occurrence of material objects found on earth and other planets [34]. Furthermore, this visualization certainly opens a perspective for every researcher to study more about the characteristics of the material that is on Earth and planet-planets in the Milky Way galaxy. Visualization of the refractive index and reflection of sodium metal based on Drude's theory supports the implementation of face-to-face and distance optics and astrophysics lectures in tertiary institutions [35].

This can be done by asking students to analyze the characteristics of the refractive index and reflection of sodium metal based on Drude's theory. Students are then asked to explore the mathematical equations of the refractive index and reflection of sodium metal based on Drude's theory. After the students get to know the details of the mathematical equations of the refractive index and reflection of Sodium metal based on Drude's theory, then they are asked to visualize the mathematical equations in the MATLAB software. The visualization of the refractive index and reflection of sodium metal based on Drude's theory can be a link between mathematical equations and optical and astrophysical experiments cheaply, easily, can be done anytime, anywhere. This visualization activity is also able to make it easier for students to apply the mathematical equations of optical and astrophysical phenomena they get during lectures [36].

The activity of visualizing mathematical equations of optical and astrophysical phenomena can support the remote implementation of optics and astrophysics lectures in tertiary institutions. The implementation of optics and astrophysics courses that implement this visualization activity can be carried out well if students can be actively involved in understanding mathematical equations of optical and astrophysical phenomena and virtual experiments with the help of MATLAB. Furthermore, through the visualization of the refractive index and reflection of sodium metal based on Drude's theory using MATLAB assistance, students can develop mathematical representation skills and creative visual representations. Besides, students' science process skills and critical thinking skills also improve.

This is because before carrying out the process of visualizing the refractive index and reflection of sodium metal based on Drude's theory, students need to explore the characteristics and mathematical equations of the refractive index and reflection of sodium metal based on Drude's theory in detail. With the current global condition that is being hit by the COVID-19 pandemic, astrophysics courses that implement this visualization process can support distance optics and astrophysics courses [37]. This is because the optics and astrophysics courses are conducted on a virtual project basis and students can be motivated to learn the concepts of optics and astrophysics.

5. Conclusion

In this research, the refractive index and reflection of sodium metal based on Drude's theory have been visualized using the help of MATLAB software. The MATLAB software is used to visualize the refractive index and reflection of sodium metal based on Drude's theory. MATLAB software was chosen to visualize the refractive index and reflection of
Sodium metal based on Drude's theory because it is one of the computational software that is easy to operate, capable of visualizing complex mathematical equations of optical and astrophysical phenomena, and producing graphical visualizations in two and three dimensions, so that becomes easy to use by college students or novice programmers. This visualization activity can make it easier for students to understand and implement mathematical equations of an optical and astrophysical phenomenon, especially regarding the refractive index and reflection of sodium metal based on Drude's theory into a virtual experiment. This activity of visualizing the refractive index and reflection of sodium metal based on Drude's theory can support the implementation of integrated optical and astrophysics lectures in tertiary institutions remotely. Furthermore, the results of this research can also be used as a source of reference for future researchers or students to be able to conduct physics lectures in an integrated manner between branches of physics. The results presented in this paper also provide an understanding of the activities of visualizing the refractive index and reflections of other materials, regardless of whether they are metals, liquids, or gases.

References


