

# Identifying Different Musical Instrument Sounds Using Fourier Analysis in LabVIEW

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**Abstract** – In this study, I found that the relationship between the Fast Fourier Transform (FFT) and music is more suitable than other methods. The LabVIEW program is capable of implementing the FFT in a shorter time, which is used to convert a signal from the time zone into a frequency field. FFT was used to find harmonics in musical notes, model sound waves, and define sound by breaking it into pieces. The amplitude of the sound received from the filtered analog input after FFT is applied is kept above the amplitude threshold level to prevent noise (low voice). If the frequency of the selected tuning note is in the specified frequency (despite possible deviations) range, tuning control is provided and the notation is performed. At the same time, spectral analysis can be examined thanks to FFT. The user can select the instrument and note they want on the interface screen. In the application, it was ensured to examine the notation and tuning process used in the intended standard reference data.

**Keywords** – notation, tuning, Fast Fourier Transform, amplitude level, LabVIEW.

## 1. Introduction

It is known that all sounds heard in our lives are periodic pressure changes that can be detected by the hearing organs and that the substance is in a simple mechanical irregularity in all forms of physical size.

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If the molecules vibrate, the sound will occur. The number of vibrations in the unit time of these vibrations is frequency [1]. The frequency of each sound has a mathematical equivalent. It is aimed to determine their frequency and amplitude by using a unique combination of mathematics and sound tones. There have been multiple studies in the literature on the use of mathematics and music together. These studies are aimed at obtaining perfect music over time. Nowadays, Pythagoras is known for his work on the theory of the right triangle.

According to Pythagoras, the universe consists of a number of harmonies. Pythagoras believed that in numbers, every unknown could be predicted and measured. Pythagoras solved the relationship of music with mathematics and transformed it into a written state to ensure the accurate transfer of music from generation to generation [2]. Pythagoras' musical melodies in a written form made the works more permanent. This allows a work that has been heard for the first time to be poured into long notes, allowing it to be studied and played later.

The foundations of the notation were laid by the philosopher Boethius and Guido of Arezzo, an Italian monk and a music theorist. Guido, who tried to find a solution to make the music lessons he gave in the monastery more practical and memorable, laid the foundations of notating using the counterpoint method. Counterpoint is a religious multi-voiced concept used after the 9<sup>th</sup> and 10<sup>th</sup> centuries. It is the art of overlapping melodic lines. With the help of the first syllables of verses in the hymn, Guido's named other notes using his fingers except the note 'B'. Guido's notated hand picture is shown in Figure 1.

Guido's 7<sup>th</sup> Note, which he called B, has long been used in the same way and was named 'B' inspired by the initials of S. Johannes in the 13<sup>th</sup> century [3]. The invention of notation has greatly benefited the development of the music world. Thanks to the notating, people who are engaged in music were able to produce works with the sound they heard on their own. In this way, they both enriched the number of works and benefited from the diversification of the music.



Figure 1. Guido's notated hand picture

In 1822, Jean Baptiste Joseph Fourier separated the sinusoidal components of a signal to obtain its frequencies through Fourier Analysis [4]. Dr. Jason Brown, a mathematician at Dalhousie University in Canada, used Fourier Analysis to convert music signals into frequencies and amplitude [5]. Nowadays, Fourier Transform (FT) is used in many applications such as signal processing, information processing, and vibration analysis. People who are engaged in music want to know the frequency at which a music signal is in order to adjust the tune. In today's developing technology, in order to obtain the frequencies of musical notes in applications used in a computer environment, and the other one can do it with comparison methods by creating a method of classification, clustering method, and a database [6], [7], [8].

## 2. National Instruments LabVIEW

In order to examine the measurements graphically or visually in the project, and to see the changes a program aimed at this goal is needed. For this reason, the LabVIEW program was used. Developed by National Instruments in 1986, it is a visual programming language interface that is preferred by engineers and scientists because of its ease compared to other programs [9]. Unlike text-based programs, a more visual environment is aimed at using the G language. LabVIEW, used for measurements, tests, data processing, and tracking, is used by connecting the measurement and circuit elements on the interface screen. The front panel and block diagram consist of two parts.

### 2.1. Front Panel

It is the interface of the user. Inputs and outputs of the projects, which are to be created, are seen in the front panel. The controls and indicators placed here are also automatically placed on the block diagram.

### 2.2. Block Diagram

The block diagram is where the actual operations take place. While the users perform the controls on the front panel, circuit elements work in the form of simulations in the block diagram. In short, the block diagram part is the G-language code writing section and the front panel defines it as a visual interface for users.

## 3. Fourier Transform (FT)

In fact, FT consists of a different method of interpreting the universe. Almost everything in the world can be defined by a waveform. Using this waveform, spectral analyses can be performed that is much more intuitive and easy to understand than the first domain view. FT is used in many fields such as electromagnetic fields, sound processing, image processing, vibration analysis, and economic calculations, and provides a unique and effective way to visually examine waveforms [10], [11]. Jean Baptiste Joseph Fourier defined the Fourier Transform Equation for the first time in his book 'Analytical Theory of Heat' written in 1822. FT mathematical form is given in equation 1 [12]:

$$S_x(f) \int_{-\infty}^{+\infty} x(t).e^{-j2\pi ft} dt \quad (1)$$

In this way, Joseph Fourier discovered that any signal can be expressed by a series of sine waves of different amplitude and phase. The sample representation is shown in Figure 2. In the frequency field, the output of the FT is expressed as "S<sub>x</sub>(f)" "x(t)" is a time zone function. "2πf" is the frequency in radians per second.

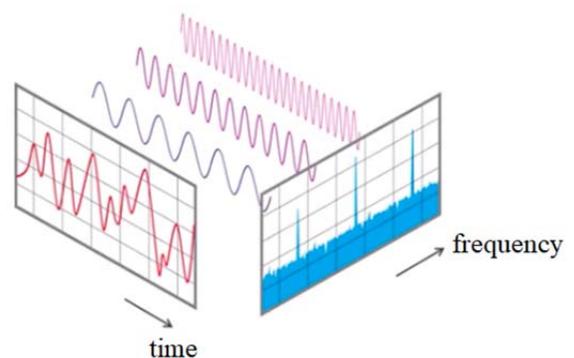


Figure 2. Wave series

In Figure 2, it can be seen that a complex-looking waveform in the time field can be represented by three vertical lines in the frequency field. This simple representation in the frequency field helps to identify the basic frequencies. Numerically, a signal can be defined as the function of one or more arguments. That is, signals are defined as functions that are bound to one or more variables that carry information about a physical state [13]. Simple periodic events in nature can be numerically defined by sinus and cosine functions. Each of the physical events is a simple periodic event [14]. If the event is repeated 'n' times per second, the function that shows simple vibrations can be expressed as a trigonometric component. An audio signal is formulated as in equation 2 as a sum of sinusoidal frequencies with different amplitudes and different frequencies [15]:

$$\sum_{i=1}^N A_i(t) \cdot \sin[2\pi F_i(t) + \theta_i(t)] \quad (2)$$

In equation 2,  $(t)$  is defined as the amplitude of the sinusoidal wave,  $F_i(t)$  is defined as the frequency of the sinusoidal wave and  $\theta_i(t)$  is defined as phase set of a sinusoidal wave. FT process is an important process used to process signals so that information from the signal can be extracted. Thus, the information obtained with the help of the FT is converted to a data format suitable for reusing. A signal applied to the FT can be specified as the sum of the basic expressions of cosine and sinus in different amplitude, frequencies, and phases. Signals can be expressed in discrete time or continuously. In other words, they can be processed digitally or as analog. The FT is calculated in two ways.

If the function is not continuous, that is, if the function consists of values belonging to certain time intervals, it can be defined as a series of sinusoidal functions at discrete frequencies. The specified frequencies are in the form of higher multiples of the original frequency. Thus, the Fourier series is obtained by collecting these sinusoidal signals. If the function is continuous, that is, defined in the actual cluster of numbers, the specified function may be reduced to the Fourier integral of the sinusoidal signals at all frequencies. FT cannot be expressed in series or integral. In non-continuous functions, it is a list of stages that make up the Fourier series, depending on discrete frequencies. In continuous functions, it is a function that depends on the frequency resulting from the receipt of the integration of the FT [16].

In the FT, it is necessary to calculate two numbers for each frequency, regardless of the method. These two numbers can be amplitude and phase values, or there can be two different numbers that carry the same information. The specified numbers can be defined by a complex number. The actual parts of

complex numbers are used to obtain information related to the frequency of the signal, while the imaginary part is used to obtain data related to phase information. This representation is generally preferred because it offers solutions for complex algebra operations. With the development of computers and programs, new methods for Fourier analysis have emerged. As a result of the studies, the program is known as "Fast Fourier Transform (FFT)" emerged. FFT saves time by reducing the number of multiplications. FFT can take a high number of samples at equal intervals on the signal [17]. The number of samples and the number of multiplications required for analysis are directly proportional. Thus, if the number of samples is halved, the number of multiplications will also be halved. FT can be done in two different ways, continuously and discretely.

### 3.1. Continuous Time Fourier Transform (CTFT)

FT and FFT map an object between two orthogonal spaces. Continuously Timed Fourier Transform (CTFT) equations are shown in equation 3 [18]:

$$\begin{aligned} F(k) &= \frac{1}{2\pi} + \int_{-\infty}^{+\infty} f(x) \cdot e^{-ikx} dx \\ f(x) &= \frac{1}{2\pi} + \int_{-\infty}^{+\infty} F(k) \cdot e^{ikx} dk \\ X(e^{j\omega}) &= \int_{-\infty}^{+\infty} X_c(t) \cdot e^{-j\omega t} dt \end{aligned} \quad (3)$$

### 3.2. Discrete Time Fourier Transform (DTFT)

When FT is applied to a discrete-time signal, the frequency spectrum of the signal is reached. When the frequency spectrum of a signal is examined, the frequency component with density can be found in the signal. The Discrete-Time Fourier Transform (DTFT) of a discrete-time signal can be expressed in equation 4:

$$X(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\Omega n} \quad (4)$$

The Fourier spectrum of a signal can be expressed as in equation 5 with amplitude and phase parts.

$$X(e^{j\Omega}) = |X(e^{j\Omega})| e^{j\angle X(e^{j\Omega})} \quad (5)$$

The amplitude spectrum of FT is defined  $|X(e^{j\Omega})|$  and the phase spectrum of FT is defined as  $\angle X(e^{j\Omega})$  [19]. When  $[n]$  is a true discrete-time signal, the amplitude spectrum becomes a double function. In this case, the phase spectrum will be a single function. When taken in the frequency spectrum  $(-\pi, \pi)$  range of the signal, the amplitude spectrum is

formed symmetrically and the phase spectrum is inversely symmetrical. Therefore, it will be enough to examine the frequency spectrum (0,  $\pi$ ) in the range. This situation is due to the symmetrical property of the DTFT. Delaying the signal over time will not create any change in the amplitude spectrum of the signal. However, it will create a linear phase shift with the amount of displacement in the phase spectrum of the signal.

In practice, when a signal needs to be displayed with DTFT; As the number of samples taken from the spectrum of the signal increases, that is, the  $N$  value used for the DTFT account is increased, the discrete frequency spectrum obtained and the continuous frequency spectrum of the signal will be more similar. The disadvantage of this situation is the increase in the amount of theoretical calculation that needs to be done. When applying the Reverse FT, the  $N$  value has to be selected large enough for the return to the original signal to occur as it should. The FFT allows for quick and easy calculation of discrete FT. The algorithm developed by Cooley and Tukey reduced the amount of processing required for DTFT [20]. In this way, the use of DTFT applications in numerical signal processing and other fields has become widespread. In addition, it has helped to develop other algorithms. All these algorithms are called FFT.

#### 4. Band-Pass Filter

A band-passing filter is a filter that transmits frequencies within a certain range and weakens frequencies outside that range. It is known that 2 parameters are needed to use the band-passing filter. The first is the determination of two limit frequencies,  $f_L$  and  $f_H$ , according to the specified base frequency. The second is the determination of the base frequency ( $f_0$ ) based on the center. In an ideal band passing filter, the middle of  $f_L$  and  $f_H$  frequencies is selected as the basic frequency [21].

It is formed by connecting one high-passing filter circuit and one low-passing circuit in the circuit of the band-passing filter. It reveals the frequency response according to the specified cutting frequencies. It will suppress frequencies other than the lower and upper cutting frequencies by cutting them. Thus, more accurate results can be obtained by filtering unwanted frequencies. The operating range of the band-pass filter is shown in Figure 3.

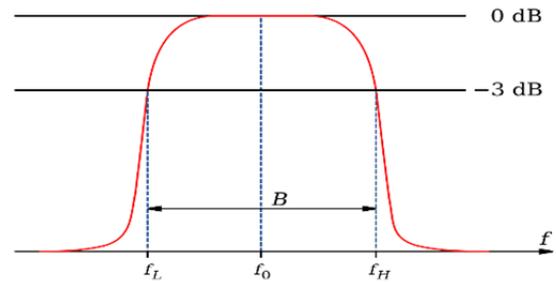


Figure 3. Band-pass filter signal diagram

### 5. Musical Instruments

#### 5.1. Violin

The violin is a string instrument, each of which gives a sound on a different screen, and has four strings. Tune sounds consist of G, D, A and E notes from low pitch sound to high pitch sound. The instrument's tuning notes of the violin is shown in Figure 4.

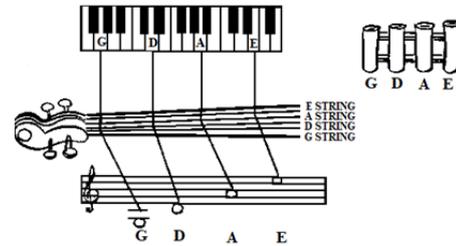


Figure 4. Tuning notes of violin

#### 5.2. Soprano Ukulele

Soprano Ukulele is a small four-string guitar. In standard classical guitars, the ukulele is different, although its strings are sorted from top to bottom from thick to thin. The thicknesses of the top wire G and the lowest wire A in the ukulele are very close to each other, and the top wire is tuned to the thin G. The tuning notes are A, E, C and G, respectively. The instrument's tuning notes of Soprano Ukulele instrument is shown in Figure 5.

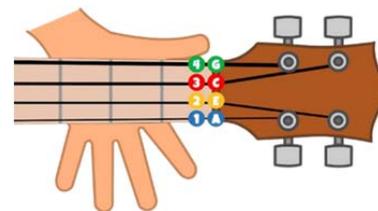


Figure 5. Tuning notes of soprano ukulele

### 5.3. Viola

It is a string musical instrument. It is very similar to the violin in its appearance and physical structure. But it is a little bigger than the violin. The viola also has four wires. But the tuning notes are different. Pesten is C, G, D and A towards the treise. The instrument's tuning notes of Viola is shown in Figure 6.

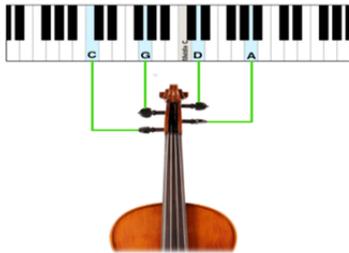


Figure 6. Tuning notes of viola

## 6. Case Application Study

### 6.1. Problem Definition and Solution

At the stage of defining the problem, the frequency values of the notes of the instrument which are to be used first were determined. Then, the stage of notation and tuning started. It will be tuned and the frequency measurements of the notes have been made. The maximum value of the FFT signal was

### 6.2. Design of Block Diagram in LabVIEW

Audio signal input is provided from an analog input (microphone) with a sampling rate of 11025 Hz. The length of the FFT is also sized by determining the same sampling rate. The basic frequency amplitude values of the lower and upper

obtained by applying FFT to the values obtained from the band-pass filter.

This obtained value was compared with the frequency measurement values.

If the frequency values obtained as a result of the comparison are high, the LED lights are green, and if it is low, it turns red. In this way, the tuning and notation process was carried out with the help of indicators.

The flowchart diagram of defining and solving the problem is shown in Figure 7.

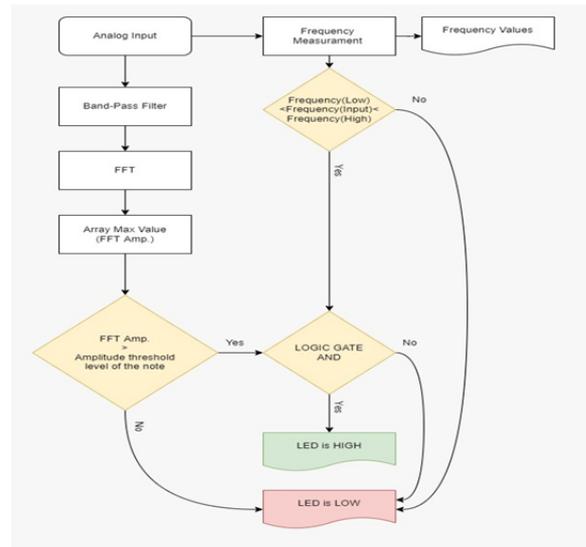


Figure 7. Flowchart diagram for problem-solving

notes close to the base note were determined to be dampened by the band-passing filter. The frequency value ranges determined for different musical instruments are given in Table 1.

Table 1. Tuning fundamental frequencies

| Instruments     | Fundamental Frequencies |                  |                  |                  |
|-----------------|-------------------------|------------------|------------------|------------------|
| Violin          | G3                      | D4               | A4               | E5               |
|                 | 196 Hz                  | 293,66 Hz        | 440 Hz           | 659,25 Hz        |
|                 | $F_L - F_H$             | $F_L - F_H$      | $F_L - F_H$      | $F_L - F_H$      |
|                 | 190,5-201,83 Hz         | 285,42-302,4 Hz  | 427,65-453,08 Hz | 640,75-678,86 Hz |
| Soprano Ukulele | A4                      | E4               | C4               | G4               |
|                 | 440 Hz                  | 329,63 Hz        | 261,63 Hz        | 392 Hz           |
|                 | $F_L - F_H$             | $F_L - F_H$      | $F_L - F_H$      | $F_L - F_H$      |
|                 | 427,65-453,08 Hz        | 320,38-339,43 Hz | 254,29-269,41 Hz | 381-403,65 Hz    |
| Viola           | C3                      | G3               | D4               | A4               |
|                 | 130,81 Hz               | 196 Hz           | 293,66 Hz        | 440 Hz           |
|                 | $F_L - F_H$             | $F_L - F_H$      | $F_L - F_H$      | $F_L - F_H$      |
|                 | 127,14-134,7 Hz         | 190,5-201,825 Hz | 285,42-302,4 Hz  | 427,65-453,08 Hz |

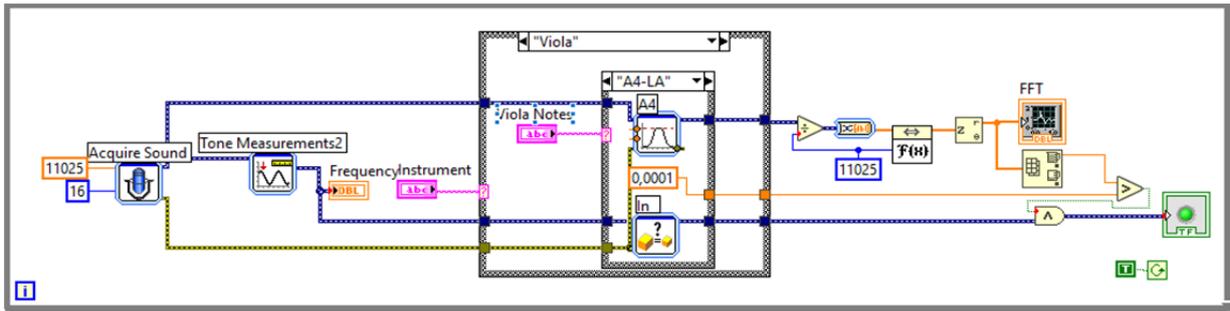


Figure 8. Block diagram

The values of the specified frequency ranges are processed into the tape-passing filters in the case structure. At the same time, the names of instruments and notes are processed. FFT has been applied to the filtered signal. The threshold amplitude value was determined to suppress the bottom sound. The amplitude value at the output of the FT was compared with the threshold amplitude value. If the amplitude value in the FFT output is greater than the threshold amplitude value, the amplitude value of the desired note has been reached. Otherwise, it is understood that the selected note does not meet the desired requirement because the amplitude value has not been reached. At the same time, the frequency of the audio signal from the analog input was measured. When reaching the frequency information of the specified note, the frequency range was determined to be 2 Hz above and below the base frequency in order to prevent possible deviations that may occur. It was checked in the case of structure whether the entrance selected in the front panel was within the

desired frequency range according to the selected tuning note.

If the controlled frequency information is within the desired range, it is understood that the frequency information of the selected note has been reached. If the frequency value is not within the desired range, it is understood that the desired condition is not met. If the benchmark and controlled frequency range at the FT output meets the desired conditions, it is confirmed with the logic gate and observed at the Boolean output. The design of the block diagram is shown in Figure 8.

### 6.3. Design of Front Panel (GUI)

The front panel is located on the top of the LED indicator designed so that the user can select the instrument. Located next to the LED indicator in order to tune the selected instrument, the notes known in the literature are presented in such a way that the user can choose. The graphical user interface is shown in Figure 9.

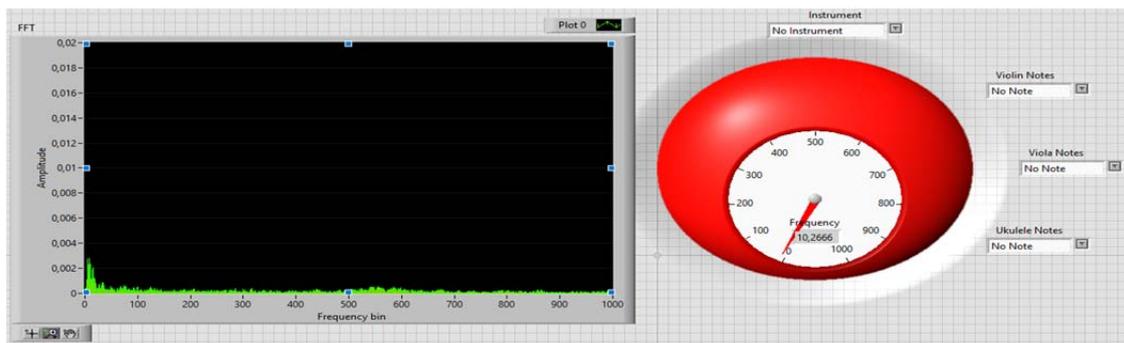


Figure 9. Graphical user interface

The note which is to be tuned has to be played until the LED indicator is green when selected by the user. If the green LED is on, the tuning will be correct. In this case, the note of the analog sound received will also be the selected tuning note. If we are facing a red LED display, we are still not on the correct note and the tuning will not have occurred. With these cases, the spectral examination can be performed thanks to the FFT indicator added for each situation. In order to perform FFT spectrum analysis,

the "no note" option ensures the observation of other unspecified frequencies. The stability of spectrum analysis is observed by the indicator of the frequency measurement added to the filter output.

### 6.4. Program Output Panel

In this study using the LabVIEW program, the notation (tuning) of Fourier analysis-based instruments was performed. Tuning is required to get

the right sound or note from the instruments. In Figure 10, the spectral analysis image is given when the same notes are selected for each instrument and audio inputs are provided.

The microphone is electronic audio equipment that in nature evolves basic sounds carried in the form of sound waves into digital sound signals thanks to air particles. Thanks to the filter applied to the sound information received from the microphone, amplitudes at unwanted frequencies were absorbed. Amplitude information that can block bottom sound is obtained by applying FFT to the sound information in the unabsorbed frequency range. At the same time, the frequency of the audio information received from the microphone was measured. If the result of this measurement is equal to or close to the fundamental frequency of the note to be tuned and the amplitude

of the FFT result is above the bottom noise, correct sound information is reached.

From the selection tab on the LED indicator, one of the 3 defined musical instruments is selected. The note next to the LED indicator, which will be tuned from the selection tab of the selected instrument, is selected. It has been observed that the LED indicator turns green within the correct sound information continuity taken from the microphone. In this case, it has been determined that FFT spectral analysis can be performed on the correct sound information when the program is stopped. If the LED indicator continues to glow red despite the selected instrument and chord note, the correct sound note has not been reached. Spectral analysis of different sound information can be performed by selecting "No tuning" from the instrument selection tab or the tuning note selection tab of the selected instrument.

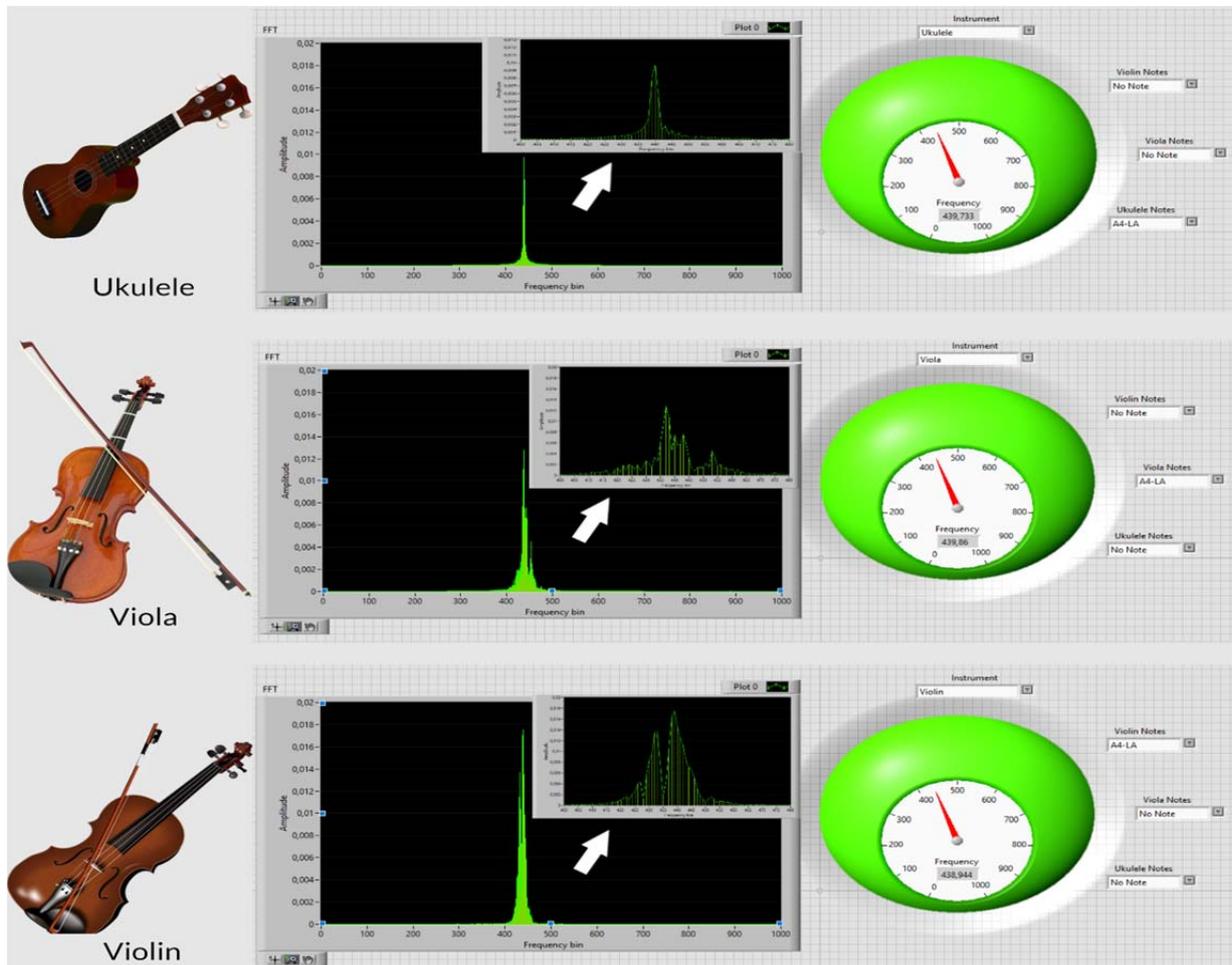


Figure 10. Test of common notes on all instruments

## 7. Conclusions

In the study, the frequency component value of the signal coming from the environment is taken in real-time and compared with the frequency values of the notes in the database prepared beforehand in the LabVIEW program. As a result of the comparison, the distance of the detected frequency from the center

frequency of the note in the database is visually reported to the user. The study, in its current form, provides simple and fast tuning for users who have just started music and whose ear for music has not yet fully developed. The people who will use this application should choose the frequency response of the microphone they will use, that is, the frequency response range while noting any instrument.

Attention should be paid to the sound pollution and acoustics of the environment to be tuned.

LabVIEW is a simulation program that provides a virtual laboratory environment. LabVIEW works using MathScript codes. The codes written in the command window are compiled and run. LabVIEW MathScript codes show a similar structure to general high-level language codes. This simulation environment offers realistic results with different measuring elements. Thanks to its large library, many test and measurement applications can be carried out. Readers may perform the notating and tuning of other instruments with the method used in this study in the future. A function can be used to generate a Chromagram from the FFT audio file obtained with the audio signal samples.

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